Assessing the foundation of the Trojan Horse Method

C.A. Bertulani

Transfer reactions
Transfer reactions

Consider the reaction

\[ A + (x+b) \rightarrow b + c + C \]
Transfer reactions

Consider the reaction

\[ A + (x+b) \rightarrow b + c + C \]

bound \( a \)

Need to solve (neglect anti-symmetrization)

\[
\left[ E - \left( K_x + K_b + K_A + V_{xb} + U_{xA} + U_{bA} \right) \right] \left| \Psi_{3B}^{(+)} \Phi_A \phi_a \right> = 0
\]

\[
\left| \Psi_{3B}^{(+)} \right> = \text{full 3-body } x+b+A \text{ wavefunction}
\]
A brief history - 1

Optical Background Representation
A brief history – 1

Optical Background Representation

• “Modification of Hauser-Feshbach calculations by direct-reaction channel coupling”
  Kawai, Kerman, McVoy, Ann. Phys. 75, 156 (1973)
A brief history - 1

Optical Background Representation

• “Modification of Hauser-Feshbach calculations by direct-reaction channel coupling”
  Kawai, Kerman, McVoy, Ann. Phys. 75, 156 (1973)

• “Fluctuations in two-step reactions through doorways”
  Kerman, McVoy, Ann. Phys. 122, 197 (1979)
A brief history - 1

Optical Background Representation

• “Modification of Hauser-Feshbach calculations by direct-reaction channel coupling”
  Kawai, Kerman, McVoy, Ann. Phys.75, 156 (1973)

• “Fluctuations in two-step reactions through doorways”
  Kerman, McVoy, Ann. Phys. 122, 197 (1979)

DWBA – Prior form
A brief history - 1

**Optical Background Representation**

- “Modification of Hauser-Feshbach calculations by direct-reaction channel coupling”

- “Fluctuations in two-step reactions through doorways”

**DWBA – Prior form**

- “The break-up of the deuteron and stripping to unbound states”
A brief history - 1

Optical Background Representation

• “Modification of Hauser-Feshbach calculations by direct-reaction channel coupling”
  Kawai, Kerman, McVoy, Ann. Phys. 75, 156 (1973)

• “Fluctuations in two-step reactions through doorways”
  Kerman, McVoy, Ann. Phys. 122, 197 (1979)

DWBA – Prior form

• “The break-up of the deuteron and stripping to unbound states”

• “Fragmentation processes in nuclear reactions”
A brief history - 2

DWBA – Post form
A brief history - 2

**DWBA – Post form**

- “Derivation of breakup-fusion cross sections from the optical theorem” Udagawa, Tamura, *PRC 24, 1348 (1981)*
A brief history - 2

**DWBA – Post form**

- “Derivation of breakup-fusion cross sections from the optical theorem”
  Udagawa, Tamura, *PRC* 24, 1348 (**1981**)  

- “Formulation of elastic and inelastic breakup-fusion reactions”
  Udagawa, Tamura, *PRC* 33, 494 (**1986**)
A brief history - 2

**DWBA – Post form**

- “Derivation of breakup-fusion cross sections from the optical theorem”
  Udagawa, Tamura, *PRC* 24, 1348 *(1981)*

- “Formulation of elastic and inelastic breakup-fusion reactions”
  Udagawa, Tamura, *PRC* 33, 494 *(1986)*

- “Exact and approximate sum rules for inclusive breakup reactions”
  Udagawa, Tamura, Mastroleo, *PRC* 37, 2261 *(1988)*
A brief history - 2

DWBA – Post form

• “Derivation of breakup-fusion cross sections from the optical theorem”
  Udagawa, Tamura, PRC 24, 1348 (1981)

• “Formulation of elastic and inelastic breakup-fusion reactions”
  Udagawa, Tamura, PRC 33, 494 (1986)

• “Exact and approximate sum rules for inclusive breakup reactions”
  Udagawa, Tamura, Mastroleo, PRC 37, 2261 (1988)

• “Equivalence of post and prior sum rules for inclusive breakup reactions”
  Ichimura, Austern, Vincent, PRC 32, 431 (1985); PRC 34, 2326 (1986);
  PRC 37, 2264 (1988)
A brief history – 2

DWBA – Post form

• “Derivation of breakup-fusion cross sections from the optical theorem”
  Udagawa, Tamura, PRC 24, 1348 (1981)

• “Formulation of elastic and inelastic breakup-fusion reactions”
  Udagawa, Tamura, PRC 33, 494 (1986)

• “Exact and approximate sum rules for inclusive breakup reactions”
  Udagawa, Tamura, Mastroleo, PRC 37, 2261 (1988)

• “Equivalence of post and prior sum rules for inclusive breakup reactions”
  Ichimura, Austern, Vincent, PRC 32, 431 (1985); PRC 34, 2326 (1986);
  PRC 37, 2264 (1988)

• “Inclusive projectile fragmentation in the spectator model”
A brief history - 2

**DWBA – Post form**


A brief history - 3

**CDCC**

- “Continuum-discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions”
A brief history - 3

CDCC

• “Continuum-discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions”

Trojan horse
A brief history - 3

CDCC

• “Continuum-discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions”

Trojan horse

• “Breakup reactions as an indirect method to investigate low-energy charged-particle reactions relevant for nuclear astrophysics”
A brief history - 3

CDCC

• “Continuum-discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions”

Trojan horse

• “Breakup reactions as an indirect method to investigate low-energy charged-particle reactions relevant for nuclear astrophysics“

• Contrib. to “Problems of Fundamental Modern Physics II”,
  Spitaleri, World Sci. 21 (1991); Spitaleri et al., PAN 74, 1763 (2011)
A brief history - 3

**CDCC**

- “Continuum-discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions”

**Trojan horse**

- “Breakup reactions as an indirect method to investigate low-energy charged-particle reactions relevant for nuclear astrophysics“


- “Indirect techniques in nuclear astrophysics”
A brief history - 3

**CDCC**

- “Continuum-discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions”

**Trojan horse**

- “Breakup reactions as an indirect method to investigate low-energy charged-particle reactions relevant for nuclear astrophysics“


- “Indirect techniques in nuclear astrophysics”
Trojan Horse versus Surrogate Reactions
Trojan Horse versus Surrogate Reactions

**Trojan Horse Method:**
Specific direct reaction induced by a tertiary beam

See, e.g. $^{12}\text{C}+^{12}\text{C}$ fusion with THM: Tumino et al, Nature 2018
Trojan Horse versus Surrogate Reactions

**Trojan Horse Method:**
Specific direct reaction induced by a tertiary beam
See, e.g. $^{12}\text{C}+^{12}\text{C}$ fusion with THM: Tumino et al, Nature 2018

**Surrogate Reaction Method:**
Relies on specific compound nuclear reaction induced by a tertiary beam
Trojan Horse versus Surrogate Reactions

Trojan Horse Method:
Specific direct reaction induced by a tertiary beam
See, e.g. $^{12}$C+$^{12}$C fusion with THM: Tumino et al, Nature 2018

Surrogate Reaction Method:
Relies on specific compound nuclear reaction induced by a tertiary beam

General theory: Inclusive breakup

- “Faddeev and DWBA description of inclusive break-up and incomplete fusion reactions”
Hussein, Frederico, Mastroleo, NPA 511, 269 (1990)
Trojan Horse versus Surrogate Reactions

Trojan Horse Method:
Specific direct reaction induced by a tertiary beam
See, e.g. $^{12}\text{C} + ^{12}\text{C}$ fusion with THM: Tumino et al, Nature 2018

Surrogate Reaction Method:
Relies on specific compound nuclear reaction induced by a tertiary beam

General theory: Inclusive breakup

• “Faddeev and DWBA description of inclusive break-up and incomplete fusion reactions”
  Hussein, Frederico, Mastroleo, NPA 511, 269 (1990)

• “Relation among theories of inclusive breakup reactions”
  M. Ichimura, PR C 41, 834 (1990)
Trojan Horse versus Surrogate Reactions

Trojan Horse Method:
Specific direct reaction induced by a tertiary beam
See, e.g. $^{12}C + ^{12}C$ fusion with THM: Tumino et al, Nature 2018

Surrogate Reaction Method:
Relies on specific compound nuclear reaction induced by a tertiary beam

General theory: Inclusive breakup

• “Faddeev and DWBA description of inclusive break-up and incomplete fusion reactions”
Hussein, Frederico, Mastroleo, NPA 511, 269 (1990)

• “Relation among theories of inclusive breakup reactions”
M. Ichimura, PR C 41, 834 (1990)
Trojan horse method
Trojan horse method

Measuring

\[ A + (b+x) \rightarrow b + c + C \]

\[
\begin{align*}
A + x & \rightarrow C + c \\
\text{(astrophysics)}
\end{align*}
\]
Trojan horse method

\[ A + (b+x) \rightarrow b + c + C \]

\[ \Rightarrow A + x \rightarrow C + c \]  
(astronomy)

\[ \sigma_{xA \rightarrow cC} = \frac{\pi}{k_x^2} \sum_l (2l+1) \left| S_{lc} \right|^2 \sim \frac{S_0}{E} e^{-2\pi \eta(E)} \]  
(astronomy)

**Measuring**

- **Trojan horse method**
- **Measuring**
- **Measuring**
Trojan horse method

Measuring

\[ A + (b+x) \rightarrow b + c + C \]

\[ \Rightarrow A + x \rightarrow C + c \]

(astrophysics)

\[ \sigma_{xA\rightarrow cC} = \frac{\pi}{k_x^2} \sum_l (2l+1) |S_{lc}|^2 \sim \frac{S_0}{E} e^{-2\pi\eta(E)} \]

(Trojan horse)

\[ \frac{d^3\sigma}{d\Omega_b d\Omega_c dE_b} = \frac{m_a m_b m_c k_b k_c}{(2\pi)^5 \hbar^6} \left| \sum_l T_{lm}(k_a, k_b, k_x) S_{lc} Y_{lm}(k_c) \right|^2 \]
Trojan horse method
Trojan horse method

Post-form DWBA

\[ T_{lm} = \left\langle \chi_b^{(-)} \psi_{xA}^{(-)} \mid V_{bx} \mid \psi_{bxA}^{(+)} \phi_a \right\rangle \approx \left\langle \chi_b^{(-)} \chi_x^{(-)} \mid V_{bx} \mid \chi_a^{(+)} \phi_a \right\rangle \]
Trojan horse method

Post-form DWBA

\[ T_{lm} = \left\langle \chi_b^{(-)} \Psi_{xA}^{(-)} | V_{bx} | \Psi_{bxA}^{(+)} \phi_a \right\rangle \approx \left\langle \chi_b^{(-)} \chi_x^{(-)} | V_{bx} | \chi_a^{(+)} \phi_a \right\rangle \]

surface-dominated \ x+A
w.f.

\[ k_x \to 0 \ (\eta \to 0) \]

\[ \chi_x^{(-)} \sim \frac{1}{k_x R} Y_{lm} e^{\pi\eta} K_{2l+1} \left( \sqrt{\frac{8R}{a_B}} \right) \]

\[ S_{le} \sim e^{-\pi\eta} \]
Trojan horse method

Post-form DWBA

\[ T_{lm} = \langle \chi_b^{(-)} \Psi_{xA}^{(-)} \mid V_{bx} \mid \Psi_{bxA}^{(+)} \phi_a \rangle \approx \langle \chi_b^{(-)} \chi_x^{(-)} \mid V_{bx} \mid \chi_a^{(+)} \phi_a \rangle \]

surface-dominated \( x+A \) w.f.

\[ k_x \rightarrow 0 \ (\eta \rightarrow 0) \]

\[ \chi_x^{(-)} \sim \frac{1}{k_x R} Y_{lm} e^{\pi \eta} K_{2l+1} \left( \sqrt{\frac{8R}{a_B}} \right) \]

\[ S_{lc} \sim e^{-\pi \eta} \]
Trojan horse method

Post-form DWBA

\[ T_{lm} = \left\langle \chi_b^{(-)} \Psi_{xA}^{(-)} \middle| V_{bx} \middle| \Psi_{bxA}^{(+)} \phi_a \right\rangle \approx \left\langle \chi_b^{(-)} \chi_x^{(-)} \middle| V_{bx} \middle| \chi_a^{(+)} \phi_a \right\rangle \]

surface-dominated \( x+A \)

w.f.

\[ k_x \to 0 \quad (\eta \to 0) \]

\[ \chi_x^{(-)} \sim \frac{1}{k_x R} Y_{lm} e^{\pi \eta} K_{2l+1} \left( \sqrt{\frac{8R}{a_B}} \right) \]

\[ S_{lc} \sim e^{-\pi \eta} \]

\[ \frac{d^3 \sigma}{d\Omega_b d\Omega_c dE_b} \to \text{const.} \quad \text{(Trojan horse)} \]


Trojan horse method: quasi-free kinematics
Trojan horse method: quasi-free kinematics

Quasi-free mechanism

\[ \frac{d\sigma}{dE_c d\Omega_c d\Omega_C} \propto KF \left| \phi(k_{bx}) \right|^2 \frac{d\sigma^{HOES}}{d\Omega_{cm}} \]

b acts as a spectator: small contribution to the reaction
Trojan horse method: quasi-free kinematics

\[
\frac{d\sigma}{dE_c \, d\Omega_c \, d\Omega_C} \propto KF \left| \phi(k_{bx}) \right|^2 \frac{d\sigma_{\text{HOES}}}{d\Omega_{\text{cm}}}
\]

Quasi-free mechanism

b acts as a spectator: small contribution to the reaction

Same reaction, using \(^3\text{He}\) (filled circles) and deuteron (open circles) as Trojan horse.


Pizzone et al, PRC 83, 045801 (2011)
Inclusive Non-Elastic Breakup Cross Section (INEB)
Consider the reaction

\[ A + (b+x) \rightarrow b + c + C \]
Consider the reaction

\[ A + (b+x) \rightarrow b + c + C \]

**INEB**

\[
\frac{d\sigma}{dE_b \, d\Omega_b} = \rho_b(E_b) \, \sigma_R^x
\]

\[
\sigma_R^x = -\frac{k_x}{E_x} \left\langle \hat{\rho}_x(r_x) \right| W_x(r_x) \left| \hat{\rho}_x(r_x) \right\rangle
\]

\[
\hat{\rho}_x(r_x) = \left( \chi_{x}^{(-)} \right| \Psi_{3B}^{(+)} \left\rangle
\]

**imaginary part opt. pot.**
Consider the reaction

\[ A + (b+x) \rightarrow b + c + C \]

**Inclusive Non-Elastic Breakup Cross Section (INEB)**

\[
\frac{d\sigma}{dE_b d\Omega_b} = \rho_b(E_b) \sigma^x_R
\]

\[
\sigma^x_R = -\frac{k_x}{E_x} \left\langle \hat{\rho}_x(\mathbf{r}_x) \left| W_x(\mathbf{r}_x) \right| \hat{\rho}_x(\mathbf{r}_x) \rightangle
\]

\[
\hat{\rho}_x(\mathbf{r}_x) = \left( \chi_b^{(-)} \right| \Psi^{(+)\,3B} \left( \mathbf{r}_x \right)
\]

**Ichimura, Austern, Vincent, PRC 32, 431 (1985) - post form**

\[
\left| \Psi^{(+)\,3B} \right\rangle = \left( E - K_b - U_b - K_x - U_x + i\epsilon \right)^{-1} V_{xb} \left| \phi_{bx} \chi^{(+)}_{bx} \right\rangle
\]
Inclusive Non-Elastic Breakup Cross Section (INEB)

\[ \hat{\rho}_{x}^{IAV}(r_{x}) = G_{x}^{(+)}(E_{x})\langle \chi_{b}^{(-)} | V_{bx} | \phi_{a} \chi_{a}^{(+)} \rangle \]

\[ G_{x}^{(+)}(E_{x}) = \left( E_{x} - K_{x} - U_{x} + i\epsilon \right)^{-1} \quad E_{x} = E - E_{b} \]
Inclusive Non-Elastic Breakup Cross Section (INEB)

\[ \hat{\rho}^\text{IAV}_x (r_x) = G_x^{(+)} (E_x) (\chi_b^{(-)} | V_{bx} | \phi_a \chi_a^{(+)} ) \]

\[ G_x^{(+)} (E_x) = (E_x - K_x - U_x + i\varepsilon)^{-1} \]

\[ E_x = E - E_b \]

Structure of \( W_x \)

Consider the scattering \( x+A \) with dynamics governed by \( H_x \)

open \( x+A \) channels

closed compound \( x+A \) channels

\[ P_x + Q_x = 1, \quad P_x Q_x = Q_x P_x = 0 \]

\[ P_x^2 = P_x, \quad Q_x^2 = Q_x \]
Structure of $W_x$
Structure of $W_x$

$$
\left( E_x - P_x P_x - P_x H_x P_x \right) \frac{1}{E_x - Q_x H_x Q_x} Q_x H_x P_x \right) P_x \left| \Psi^{(+)}_{xA} \right> = 0
$$

Exact but useless!
Structure of $W_x$

$$\left( E_x - P_x H_x P_x - P_x H_x Q_x \frac{1}{E_x - Q_x H_x Q_x} Q_x H_x P_x \right) P_x |\Psi^{(+)}_{xA}\rangle = 0$$

**Exact but useless!**

- **Average** over CN states
- Define optical $x+A$ open channels $|\tilde{\Psi}^{(+)}_{xA}\rangle$
- Split $P_x$ into $p^{(0)}_x$(elastic breakup of $b+x+A$) + $p^{(D)}_x$(all open non-elastic direct channels)
Structure of $W_x$

$$\left( E_x - P_x H_x P_x - P_x H_x Q_x \frac{1}{E_x - Q_x H_x Q_x} Q_x H_x P_x \right) P_x \left| \Psi^{(+)}_{xA} \right\rangle = 0$$

Exact but useless!

- **Average** over CN states
- Define optical $x+A$ open channels $\left| \tilde{\Psi}^{(+)}_{xA} \right\rangle$
- Split $P_x$ into $P^{(0)}_x$(elastic breakup of $b+x+A$) $+$ $P^{(D)}_x$(all open non-elastic direct channels)

$$\left( E_x - H^{(eff)}_x - H^{(eff)}_x P_x G^{(+),D}_x P_x H^{(eff)}_x \right) P_x \left| \Psi^{(+)}_{xA} \right\rangle = 0$$
Structure of $W_x$
Structure of $W_x$

$$W_x = \text{Im} \ U_x = W_x^{CN} + W_x^D$$

$$\text{Im} \ H_x^{(\text{eff})} \quad \text{Im} \left[ \begin{array}{c} \text{Im} \ H_x^{(\text{eff})} P_x^{(D)} G_x^{(+),D} P_x^{(D)} H_x^{(\text{eff})} \end{array} \right]$$

$$G_x^{(+),D} = \left[ E_x - P_x^{(D)} H_x^{(\text{eff})} P_x^{(D)} + i\varepsilon \right]^{-1}$$
Structure of $W_x$

$$W_x = \text{Im } U_x = W_x^{\text{CN}} + W_x^D$$

$$\text{Im } H_x^{(\text{eff})} \quad \text{Im} \left[ H_x^{(\text{eff})} P_x^{(D)} G_x^{(+,D)} P_x^{(D)} H_x^{(\text{eff})} \right]$$

$$G_x^{(+,D)} = \left[ E_x - P_x^{(D)} H_x^{(\text{eff})} P_x^{(D)} + i\varepsilon \right]^{-1}$$

Spectral decomposition

$$\text{Im } G_x^{(+,D)} = -\pi \sum_D \int \frac{dk_D}{(2\pi)^3} \left| \chi_{k_D}^{(-)} \right> \delta \left( E_x - E_{k_D} \right) \left( \chi_{k_D}^{(-)} \right)$$
Inclusive Non-Elastic Breakup Cross Section (INEB)
Inclusive Non-Elastic Breakup Cross Section (INEB)

\[
\frac{d^2 \sigma_{b}^{\text{INEB},(D)}}{dE_b d\Omega_b} = \rho_b \left( E_b \right) \frac{k_x}{E_x} \left\langle \rho_{x}^{(+)} | W^D_x | \rho_{x}^{(+)} \right\rangle
\]
Inclusive Non-Elastic Breakup Cross Section (INEB)

\[
\frac{d^2 \sigma_{\text{INEB},(D)}}{dE_b d\Omega_b} = \rho_b \left( \frac{E_b}{E_x} \right)^k \langle \rho^{(+)} XAV | W^D_x | \rho^{(+)} XAV \rangle
\]

All interactions coupling the ground to direct inelastic states

\[
W^D_x = -\pi \sum_f \frac{dk_f}{(2\pi)^3} V_{(0,f)} \left| \chi_{x}^{(-)}(k_f) \right\rangle \langle \chi_{x}^{(-)}(k_f) \right| V_{(0,f)} \delta(E_x - E_f)
\]

\[
V_{(0,f)} = P^{(0)}_x V P^{(D)}_x
\]
Inclusive Non-Elastic Breakup Cross Section (INEB)
Inclusive Non-Elastic Breakup Cross Section (INEB)

\[
\frac{d^2 \sigma_{b}^{\text{INEB},(D)}}{dE_b \, d\Omega_b} = -\pi \rho_b \left( \frac{E_b}{E_x} \right) \sum_f \frac{dk_f}{(2\pi)^3} \delta(E_x - E_f) \left| \langle \chi^(-) (k_f) | V_{(0,f)} | \rho_{x}^{(+)} \rangle \right|^2
\]
Inclusive Non-Elastic Breakup Cross Section (INEB)

\[
\frac{d^2 \sigma_{b}^{\text{INEB,(D)}}}{dE_b d\Omega_b} = -\pi \rho_b \left( \frac{E_b}{E_x} \right) \frac{k_x}{E_x} \sum_f \frac{dk_f}{(2\pi)^3} \delta(E_x - E_f) \left| \langle \chi_f^{(-)}(k_f) | V_{(0,f)} | \rho_x^{(+)\text{IAV}} \rangle \right|^2
\]

Consider a particular \( x+A \) channel, i.e., \( y+B \)

\[
\frac{d^2 \sigma_{b}^{\text{INEB,(D)}}}{dE_b d\Omega_b dE_y d\Omega_y} = -\pi \rho_b \left( \frac{E_b}{E_x} \right) \rho_y \left( \frac{E_y}{E_x} \right) \frac{k_x}{E_x} \times \left| \langle \chi_y^{(-)}(k_y) | V_{(x,y)} G_x^{(+)\text{IAV}} | \chi_b^{(-)} | V_{xb} \Phi_a \chi_a^{(+)} \rangle \right|^2
\]

transition to \( y \) channel  propagation of \( x \)  elastic breakup
The Trojan Horse formula
The Trojan Horse formula

Assume factorization

\[ \chi_a^{(+)}(r_b, r_x) = \chi_b^{(+)}(r_b) \chi_x^{(+)}(r_x) \]
The Trojan Horse formula

Assume factorization

$$\chi_{a}^{(+)}(r_b, r_x) = \chi_{b}^{(+)}(r_b)\chi_{x}^{(+)}(r_x)$$

and define

$$\hat{S}_b(r_x) = \left\langle \chi_{(b, k_b)}^{(-)}(r_b)\phi_a(r_x, r_b) \left| \chi_{(b, k_b)}^{(+)}(r_b) \right. \right\rangle$$
The Trojan Horse formula

Assume factorization

\[ \chi^{(+)}_a(r_b, r_x) = \chi^{(+)}_b(r_b) \chi^{(+)}_x(r_x) \]

and define

\[ \hat{S}_b(r_x) = \langle \chi^{(-)}(r_b) \phi_a(r_x, r_b) | \chi^{(+)}(r_b) \rangle \]

\[ \Rightarrow \frac{d^2 \sigma^{\text{INEB,(D)}}_b}{dE_b d\Omega_b} = \rho_b(E_b) \frac{k_x}{E_x} \left| \langle \chi^{(-)}_y | V_{(x,y)} \hat{S}_b(r_x) | \chi^{(+)}_x \rangle \right|^2 \]
The Trojan Horse formula

Assume factorization

\[ \chi_a^{(+)}(\mathbf{r}_b, \mathbf{r}_x) = \chi_b^{(+)}(\mathbf{r}_b) \chi_x^{(+)}(\mathbf{r}_x) \]

and define

\[ \hat{S}_b(\mathbf{r}_x) = \left\langle \chi_{(b, k_b)}^{(-)}(\mathbf{r}_b) \phi_a(\mathbf{r}_x, \mathbf{r}_b) \right| \chi_{(b, k_b)}^{(+)}(\mathbf{r}_b) \right\rangle \]

\[ \frac{d^2 \sigma_{\text{INEB},(D)}}{dE_b \, d\Omega_b} = \rho_b(E_b) \frac{k_x}{E_x} \left| \left\langle \chi_y^{(-)} \right| V_{(x,y)} \hat{S}_b(\mathbf{r}_x) \left| \chi_x^{(+)} \right\rangle \right|^2 \]

Approximation: maximum survival probability for b

\[ \hat{S}_b(\mathbf{r}_x) \approx \phi_a(\mathbf{r}_b) \]
The Trojan Horse formula

Assume factorization

\[ \chi_{a}^{(+)}(r_b, r_x) = \chi_{b}^{(+)}(r_b) \chi_{x}^{(+)}(r_x) \]

and define

\[ \hat{S}_b(r_x) = \left\langle \chi_{(b,k_b)}^{(-)}(r_b) \phi_a(r_x, r_b) \right| \chi_{(b,k_b)}^{(+)}(r_b) \right\rangle \]

\[ \frac{d^2 \sigma_{b}^{\text{INEB,(D)}}}{dE_b d\Omega_b} = \rho_b(E_b) \frac{k_x}{E_x} \left| \left\langle \chi_{y}^{(-)} \right| V_{(x,y)} \hat{S}_b(r_x) \right| \chi_{x}^{(+)} \right\rangle^2 \]

Approximation: maximum survival probability for b

\[ \hat{S}_b(r_x) \approx \phi_a(r_b) \]

\[ \frac{d^2 \sigma_{b}^{\text{INEB,(D)}}}{dE_b d\Omega_b} = \frac{d^2 \sigma_{b}^{\text{(THM)}}}{dE_b d\Omega_b} = K_{\text{(THM)}} \left| \phi(k_b) \right|^2 \sigma_{(x+A \rightarrow y+B)} \]
Conclusions
Conclusions

• The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.
Conclusions

• The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

• The Trojan Horse quasi-free cross section for the x+A subsystem can be obtained under approximations:
Conclusions

• The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

• The Trojan Horse quasi-free cross section for the x+A subsystem can be obtained under approximations:
  - No antisymmetrization
Conclusions

- The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

- The Trojan Horse quasi-free cross section for the $x+A$ subsystem can be obtained under approximations:
  - No antisymmetrization
  - Weak influence of spectator $b$ on $x+A$
Conclusions

- The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

- The Trojan Horse quasi-free cross section for the \( x+A \) subsystem can be obtained under approximations:
  - No antisymmetrization
  - Weak influence of spectator b on \( x+A \)
  - Factorization of 3-body wavefunctions
Conclusions

• The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

• The Trojan Horse quasi-free cross section for the x+A subsystem can be obtained under approximations:
  - No antisymmetrization
  - Weak influence of spectator b on x+A
  - Factorization of 3-body wavefunctions
  - Compound nucleus formation only included as a statistical contribution to potentials and wavefunctions
Conclusions

• The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

• The Trojan Horse quasi-free cross section for the x+A subsystem can be obtained under approximations:
  – No antisymmetrization
  – Weak influence of spectator b on x+A
  – Factorization of 3-body wavefunctions
  – Compound nucleus formation only included as a statistical contribution to potentials and wavefunctions
  – Peripherality, i.e., maximum survival probability of b

Conclusions

• The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

• The Trojan Horse quasi-free cross section for the $x+A$ subsystem can be obtained under approximations:
  - No antisymmetrization
  - Weak influence of spectator $b$ on $x+A$
  - Factorization of 3-body wavefunctions
  - Compound nucleus formation only included as a statistical contribution to potentials and wavefunctions
  - Peripherality, i.e., maximum survival probability of $b$

Conclusions

• The inclusive breakup formalism contains all information about the two-body clustered projectile reacting with a target.

• The Trojan Horse quasi-free cross section for the x+A subsystem can be obtained under approximations:
  - No antisymmetrization
  - Weak influence of spectator b on x+A
  - Factorization of 3-body wavefunctions
  - Compound nucleus formation only included as a statistical contribution to potentials and wavefunctions
  - Peripherality, i.e., maximum survival probability of b