Spectroscopic Factors in the Islands of Inversion à la Nilsson

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Outline

Short Introduction

Spectroscopic Factors in the Nilsson Strong Coupling Limit

\( N=20 \) Island of Inversion

\( N=8 \) \(^{12}\text{Be}\)

Summary
“Classic” magic numbers are generally correct only for stable and near stable isotopes.

Experimental studies of new exotic isotopes revealed changes in shell structure and collectivity, and provided insight on the important role played by the central, tensor, (and 3N) forces in these changes.

Much evidence has been obtained for the existence of deformed ground states, and a good understanding of the physical mechanism behind the inversion.

Based on this fact it is of interest to consider the description of direct reactions in the rotational model framework.
Structure of $^{33}\text{Mg}$ sheds new light on the $N = 20$ island of inversion

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ABSTRACT

The first reaction spectroscopy on the ground state structure of $^{33}\text{Mg}$ through the measurement of the longitudinal momentum distribution from the one-neutron removal reaction using a C target at 898A MeV is reported. The experiment was performed at the FRS, GSI. The distribution has a relatively narrow width (150 $\pm$ 3 MeV/c (FWHM)) and the one-neutron removal cross-section is 74 $\pm$ 4 mb. An increased contribution from the $2p_{3/2}$ orbital is required to explain the observation showing its lowering compared to existing model predictions. This provides new information regarding the configuration of $^{33}\text{Mg}$ and the island of inversion.
Direct experimental evidence for a multiparticle-hole ground state configuration of deformed $^{33}$Mg

The first direct experimental evidence of a multiparticle-hole ground state configuration of the neutron-rich $^{33}$Mg isotope has been obtained via intermediate energy (400 A MeV) Coulomb dissociation measurement. The major part $\sim(70 \pm 13)\%$ of the cross section is observed to populate the excited states of $^{32}$Mg after the Coulomb breakup of $^{33}$Mg. The shapes of the differential Coulomb dissociation cross sections in coincidence
Assume ground state of $^{33}\text{Mg}$ is the $3/2[^321]$ neutron Nilsson level
$^{33}\text{Mg} - 1\text{n removal à la Nilsson}$

$^{33}\text{Mg} \rightarrow 3^{2}\text{Mg} \rightarrow 3^{1}\text{Mg}$

\[
\frac{d\sigma}{d\Omega} = \sum_{j, \ell} g^2 \langle I_i j K_i \Delta K | I_f K_f \rangle^2 C_{j, \ell}^2 \langle \phi_f | \phi_i \rangle^2 \sigma^{-1n}_{\ell}
\]

\[
= \sum_{j, \ell} S_{j, \ell} \times \sigma^{-1n}_{\ell}
\]

B. Elbek and P. Tjom,
\[ S_{j,\ell}(-1n) = 2\langle \frac{3}{2} j \frac{3}{2} | \frac{3}{2} I 0 \rangle^2 C_{j,\ell}^2 \]
\[ \mu = \frac{3}{5} \left( g_s \langle s_3 \rangle + g_R \right) \]

\[ \langle s_3 \rangle = \frac{1}{2} \left( C_{3/2,1}^2 + \frac{3}{7} \left( C_{7/2,3}^2 - C_{5/2,3}^2 \right) - \frac{4\sqrt{10}}{7} C_{7/2,3} C_{5/2,3} \right) \]
### $^{33}\text{Mg} - 1\text{n removal à la Nilsson}$

<table>
<thead>
<tr>
<th>Final state</th>
<th>Energy [MeV]</th>
<th>Experimental $S_{j,\ell}$</th>
<th>Calculated $S_{j,\ell}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>0.00</td>
<td>$0.6^{+0.3}_{-0.5}$</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.19 \pm 0.1$</td>
<td>0.24</td>
</tr>
<tr>
<td>$2^+$</td>
<td>0.89</td>
<td>$0.5^{+0.7}_{-0.3}$</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$0.5^{+0.2}_{-0.5}$</td>
<td>0.34</td>
</tr>
<tr>
<td>$4^+$</td>
<td>2.32</td>
<td>3</td>
<td>0.55</td>
</tr>
</tbody>
</table>

\[
\left| \frac{3}{2}[321] \right> = (-0.65 \pm 0.15)|p_{3/2}\rangle + (0.75^{+0.13}_{-0.23})|f_{7/2}\rangle \\
+ ( - 0.12^{+0.08}_{-0.22})|f_{5/2}\rangle
\]

I. Hamamoto, PRC 81, 021304(R) (2010)


Same Nilsson level $3/2[321]$

$I. \text{Hamamoto,}
\text{PRC 81, 021304(R) (2010)}$

$T. \text{Nakamura, et al.}
\text{PRL 112, 142501 (2014)}$

$|\frac{1}{2}[330]\rangle = C_{1/2,1}|p_{1/2}\rangle + C_{3/2,1}|p_{3/2}\rangle + C_{5/2,3}|f_{5/2}\rangle + C_{7/2,3}|f_{7/2}\rangle$
WEAK BINDING

Green (1956), Bohr & Mottelson (1969)

- Low $l$ levels ($s$, $p$) → extended wavefunctions ("halos")

$\Delta x \cdot \Delta p \sim \hbar$
$S_{3/2,1}(-1n) = 2C_{3/2,1}^2$

$S_{7/2,3}(-1n) = 2C_{7/2,3}^2$
$^{32}\text{Mg} - 1\text{n removal à la Nilsson}$

<table>
<thead>
<tr>
<th>Final state</th>
<th>Energy [MeV]</th>
<th>$\ell$</th>
<th>$S_{J\ell}$</th>
<th>Calculated $S_{J\ell}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^-$</td>
<td>0.22</td>
<td>1</td>
<td>$0.59^{+0.11}_{-0.11}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$7/2^-$</td>
<td>0.46</td>
<td>3</td>
<td>$1.24^{+0.4}_{-0.4}$</td>
<td>1.7</td>
</tr>
</tbody>
</table>

$$\frac{3}{2}[321] \approx (-0.54 \pm 0.05)|p_{3/2}\rangle + (0.79 \pm 0.13)|f_{7/2}\rangle + (-0.29 \pm 0.36)|f_{5/2}\rangle.$$  

The Nilsson Picture in $^{12}\text{Be}$

\[ \left| \frac{1}{2} [220] \right> = C_{1/2,0} \left| s_{1/2} \right> + C_{3/2,2} \left| d_{3/2} \right> + C_{5/2,2} \left| d_{5/2} \right> \]

\[ \left| \frac{1}{2} [101] \right> = C_{1/2,1} \left| p_{1/2} \right> + C_{3/2,1} \left| p_{3/2} \right> \]

A. Bohr and B. R. Mottelson, Nuclear Structure Volume II
W. Von Oertzen, M. Freer, and Y. Kanada-En’yo Physics Reports 432 (2006) 43 – 113
Direct Reactions Studies

1n Removal


J. Kahlbow et al.
A. Enriques et al. QFS workshop at York (2017)
A. Corsi et al.

(d,p)


$^{11}\text{Be}(d,p)^{12}\text{Be}$ à la Nilsson

$^{1/2^+} 1/2[220]$  

$K_i=1/2$  

$^{11}\text{Be}$

$S_{1/2}$  

$d_{3/2}$  

$d_{5/2}$

$K_f=0$  

$^{12}\text{Be}$

$|0_{2}^+\rangle = -\beta |\nu_{1} \bar{\nu}_{1}\rangle + \alpha |\nu_{2} \bar{\nu}_{2}\rangle$

$|0_{1}^+\rangle = \alpha |\nu_{1} \bar{\nu}_{1}\rangle + \beta |\nu_{2} \bar{\nu}_{2}\rangle$
Total of 12 relations connecting the experimental data to four unknown amplitudes which we determine from a chi\(^2\)-minimization procedure.

Weighted fit of the relative spectroscopic factor values with respect to the ground state transition for each of the data sets, and of the absolute value of the \(^{11}\)Be ground-state magnetic moment.
Results

\[ \left| \frac{1}{2}[220] \right\rangle \approx -0.72(4) \left| s_{1/2} \right\rangle - 0.09(2) \left| d_{3/2} \right\rangle + 0.69(4) \left| d_{5/2} \right\rangle \]

\[ \left| \frac{1}{2}[101] \right\rangle \approx 0.68(4) \left| p_{1/2} \right\rangle + 0.73(3) \left| p_{3/2} \right\rangle \]

\[ \alpha = 0.73(4) \text{ and } \beta = 0.69(4) \]
# Results

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Final state</th>
<th>Energy [MeV]</th>
<th>( \ell )</th>
<th>Experimental ( S_{i,f} )</th>
<th>Present work</th>
<th>Theoretical ( S_{i,f} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[17]</td>
<td>[23]</td>
<td>[24,25]</td>
</tr>
<tr>
<td>( ^{11})Be</td>
<td>( ^{12})Be</td>
<td>0.00</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{1}{2}^+ )</td>
<td>( 0_1^+ )</td>
<td>2.11</td>
<td>2</td>
<td>0.36(29)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{1}{2}^- )</td>
<td>( 0_2^- )</td>
<td>2.24</td>
<td>0</td>
<td>2.61(134)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( ^{10})Be</td>
<td>( ^{11})Be</td>
<td>0.00</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>#</td>
</tr>
<tr>
<td>( \frac{1}{2}^+ )</td>
<td>( 0_1^+ )</td>
<td>1.78</td>
<td>2</td>
<td>0.86(29)</td>
<td>1</td>
<td>#</td>
</tr>
<tr>
<td>( \frac{1}{2}^- )</td>
<td>( 0_2^- )</td>
<td>2.69</td>
<td>1</td>
<td>0.71(26)</td>
<td>1</td>
<td>#</td>
</tr>
<tr>
<td>( ^{11})Be</td>
<td>( ^{10})Be</td>
<td>3.4</td>
<td>2</td>
<td>1.0(2)</td>
<td>#</td>
<td>1.0(2)</td>
</tr>
</tbody>
</table>

**PVC (NFT)**

**SM**

**Core Coupling**
Predictions

$^{12}\text{Be}(d,p)^{13}\text{Be}$

$|0^+_1\rangle = \alpha \nu_1 \bar{\nu}_1 + \beta \nu_2 \bar{\nu}_2$

$K_f=0$

$S_{1/2}$

$K_i=1/2$

$\frac{1}{2}$ $\frac{1}{2}[101]$

$\frac{5}{2}$

$\frac{1}{2}$ $\frac{1}{2}[220]$

$1/2$

$\nu_1$ and $\nu_2$

$^{12}\text{Be}$

$^{13}\text{Be}$

$|\frac{1}{2}[220]\rangle = C_{1/2,0} s_{1/2} + C_{3/2,2} d_{3/2} + C_{5/2,2} d_{5/2}$

$|\frac{1}{2}[101]\rangle = C_{1/2,1} p_{1/2} + C_{3/2,1} p_{3/2}$
Predictions

\[ ^{12}\text{Be}(d,p)^{13}\text{Be} \]

TRIUMF R. Kanungo, \textit{et al.}

\[ S_{0+1/2+} = C_{1/2,0}^2 \beta^2 \]
\[ S_{0+5/2+} = \frac{1}{3} C_{5/2,2}^2 \beta^2 \]
\[ S_{0+1/2-} = C_{1/2,1}^2 \alpha^2 \]

\[ ^{13}\text{B}(d,^{3}\text{He})^{12}\text{Be} \]

RCNP C. Santamaria, \textit{et al.}

<table>
<thead>
<tr>
<th>Initial State</th>
<th>Final State</th>
<th>Energy [MeV]</th>
<th>(\ell)</th>
<th>Calculated (S_{i,f})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{12}\text{Be})</td>
<td>(^{13}\text{Be})</td>
<td>0.00</td>
<td>0</td>
<td>0.24(3)</td>
</tr>
<tr>
<td>(0^+_1)</td>
<td>(1/2^+)</td>
<td>~1.8</td>
<td>2</td>
<td>0.07(1)</td>
</tr>
<tr>
<td>(5/2^+)</td>
<td>(1/2^-)</td>
<td>0+x</td>
<td>1</td>
<td>0.25(4)</td>
</tr>
<tr>
<td>(^{13}\text{B})</td>
<td>(^{12}\text{Be})</td>
<td>0.00</td>
<td>1</td>
<td>0.50</td>
</tr>
<tr>
<td>(3^-/2)</td>
<td>(0^+_1)</td>
<td>2.11</td>
<td>1</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(2^-_1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0^+_2)</td>
<td>2.24</td>
<td>1</td>
<td>0</td>
</tr>
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</table>

Many Thanks to Antonio Moro!
Summary

• A Nilsson description of nucleon knockout and transfer reactions in the deformed ‘Islands of Inversion’ at $N=8$ and $N=20$ allows a straightforward analysis of results.

• Nilsson wavefunction amplitudes are directly related to the spectroscopic factors.

• $N=20$ – adjusted wavefunction for $3/2[321]$ level is consistent with a reduced $1f_{7/2}-2p_{3/2}$ gap, in line with SM results.

• $N=8$ – wavefunctions, derived from a minimization procedure, give very good agreement with all available data → meaningful predictions for other reactions.
Acknowledgements
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ありがとうございます
Arigato gozaimasu